Configurational entropy of critical earthquake populations

C. Goltz and M. Böse
Department of Geophysics, Kiel University, Germany

Received 24 May 2002; revised 26 July 2002; accepted 13 August 2002; published 29 October 2002.

[1] We present an approach to describe the evolution of distributed seismicity by configurational entropy. We demonstrate the detection of phase transitions in the sense of a critical point phenomenon in a 2D site-percolation model and in temporal and spatial vicinity to the 1992, M7.3 Landers earthquake in Southern California. Our findings support the assumption of intermittent criticality in the Earth's crust. We also address the potential usefulness of the method for earthquake catalogue declustering. *INDEX TERMS:* 3210 Mathematical Geophysics: Modeling; 3220 Mathematical Geophysics: Nonlinear dynamics; 3250 Mathematical Geophysics: Fractals and multifractals; 7209 Seismology: Earthquake dynamics and mechanics; 7223 Seismology: Seismic hazard assessment and prediction. **Citation:** Goltz, C., and M. Böse, Configurational entropy of critical earthquake populations, *Geophys. Res. Lett.*, *29*(20), 1990, doi:10.1029/2002GL015540, 2002.

1. Introduction

[2] The description and interpretation of spatio-temporal variability in seismicity is key to the comprehension of the earthquake process and to time dependent hazard analysis. In the past, studies in earthquake research focused only on local patterns, as e.g., characteristic Mogi donuts [*Mogi*, 1969], seismic gaps [e.g., *Fedotov*, 1965], seismic quiesence [e.g., *Wyss et al.*, 1996] or foreshocks [e.g., *Dodge et al.*, 1996]. However, the often observed increasing number of moderate earthquakes prior to a mainshock, for example, cannot be taken for a classical series of foreshocks, due to the fact that the involved areas are too large to be explained by traditional seismology [e.g., *Jones and Molnar*, 1979; *Ellsworth et al.*, 1981; *Knopoff et al.*, 1996]. Recent studies take these and other long-range correlations into account by expanding the area of interest from local to regional scale. Motivation for this change arises from the critical point concept for earthquakes which assumes that the Earth's crust is a complex nonlinear system that produces large non-random earthquakes when it is in a critical state [e.g., *Keilis-Borok*, 1990; *Sornette and Sornette*, 1990; *Sornette and Sammis*, 1995]. If the regional fault system is in a subcritical state, the occurrence of strong events is rare. When the system proceeds towards criticality, the release of a large earthquake is much more likely. The tendency to remain near the critical point corresponds to the behaviour of self-organized criticality (SOC) [*Bak and Tang*, 1989]. Fluctuations in the level of criticality, as observed in this paper by fluctuating entropy, support the competing notion of intermittent criticality [*Jaume and Sykes*, 1999]. The latter situation would imply some level of statistical predictability [*Main and Al-Kindy*, 2002, and references therein]. The critical point itself is characterised by a self-similiarity of correlations in the system. Its behaviour is defined by many different scales, meaning that characteristic quantities within the system vanish. These quantities diverge in the critical point or are limited by the size of the physical system only [e.g., *Sammis and Smith*, 1999]. The best known example for an application of the critical point concept in earthquake statistics is the so-called accelerated moment release (AMR) prior to large events [*Bufe et al.*, 1994; *Sammis et al.*, 1996; *Sykes and Jaume*, 1990; *Knopoff et al.*, 1996; *Bowman et al.*, 1998; *Jaume and Sykes*, 1999]. However, the analysis of AMR, similar to other determinations of the critical transition based on correlation lengths, e.g., using the single-link cluster analysis [*Zöller et al.*, 2001], has the disadvantage that it depends on free parameters. Below we will introduce a new method that represents the most direct physical approach and only weakly depends on very few non-critical parameters.

2. Configurational Entropy

[3] After *Clausius*' [1850] introduction of entropy in thermodynamics, followed by *Boltzmann*'s [1871] statistical formulation, the concept of entropy developed in many different branches of science. Most recently, *Main and Al-Kindy* [2002] address the proximity of the global earthquake population to the critical point by examining energy and entropy. They do not rule out the possibility of SOC but conclude that the observed temporal entropy fluctuations of the order of ±10% rather support the notion of intermittent criticality as the traditional definition of SOC would probably require less fluctuation. Configurational entropy H is derived from *Shannon*'s [1948] information theory and is a function of the probability p_i of occurrence of different states in a system. The gain in information I from the occurrence of the event i ($i = 1, 2, \ldots, n$) then is defined by Shannon as

$$I_i = \ln \frac{1}{p_i}. \tag{1}$$

The entropy H of the system gives the expected value of I, such that

$$H = \sum_{i=1}^{n} p_i I_i = -\sum_{i=1}^{n} p_i \ln p_i. \tag{2}$$

For the application of the concept of entropy to binary images with seismically active (black) and inactive (white) pixels, one may overlay the seismic pattern of dimension $L \times L$ with a regular grid of cells of size $l \times l$. The probability of occurrence of a state k ($p_k(l)$) can easily be

0094-8276/02/2002GL015540

computed from the number of cells containing k events ($N_k(l)$) and the total number of cells within the grid ($N(l)$):

$$p_k(l) = \frac{N_k(l)}{N(l)}. \tag{3}$$

Analogous to (2) the configurational entropy then is

$$H(l) = -\sum_{i=0}^{l^2} p_i(l) \ln p_i(l). \tag{4}$$

The summation from 0 to l^2 corresponds to the number of possible states, because 0 to l^2 events can occur in a cell of length l on the conditions that the space is integer discretised and that non-unique points have been removed. H will undergo its maximum H_{max} when the probability of occurrence of all possible states is equal, such that $p_i = const$ for all $i \in [0, l^2]$. It can be shown [cf. *Beghdadi et al.*, 1993] that this is only the case if

$$p_i = \frac{1}{l^2 + 1}. \tag{5}$$

Using (4) this means, that

$$H_{max}(l) = \ln\left(l^2 + 1\right). \tag{6}$$

To enable the comparison of entropy values at different scales l, we normalise H using this maximum:

$$H^*(l) = \frac{H(l)}{H_{max}(l)}. \tag{7}$$

We thus do not need to define any characteristic length (e.g., cell size) before the analysis is carried out. Problems such as a non-integer relation between L and l are avoided by using a sliding box rather than a regular grid. When applied to a random fractal dataset, H^* exhibits a maximum H^{**} at a characteristic scale l^*, the so-called optimum length (Figure 1). l^* marks the scale at which the point set has to be looked at to unveil the maximum information contained in the pattern. As earthquakes show fractal behaviour in size, time and space [see, *Bak et al.*, 2002, for a discussion and unification of these individual scaling laws] we expect well-defined maxima in earthquake patterns, too. We would like to stress that earthquake patterns are random fractals, i.e., that they are self-similar only in the statistical sense. There is more than scaling to seismicity as witnessed by the well-pronounced maximum in H^* (cf. Figure 1) which does not exist in the case of non-random processes. The scaling exponent itself is a poor parameter to detect temporal change in a fractal distribution as it, by its very nature, only describes a very general property of the set [*Goltz*, 1997]. Although H is normalised for comparison between scales, H^{**} is, by definiton, scale dependent. In summary, our method is scale independent initially but the final result is not.

3. Entropy at the Critical Point

[4] In a thermodynamical system, as e.g., in a pot of boiling water, the second order phase transition from liquid to gaseous state entails a strong and steady increase of entropy at a critical temperature T_c. To simulate a critical point process with a characteristic phase transition in a point configuration such as an epicentre distribution we make use

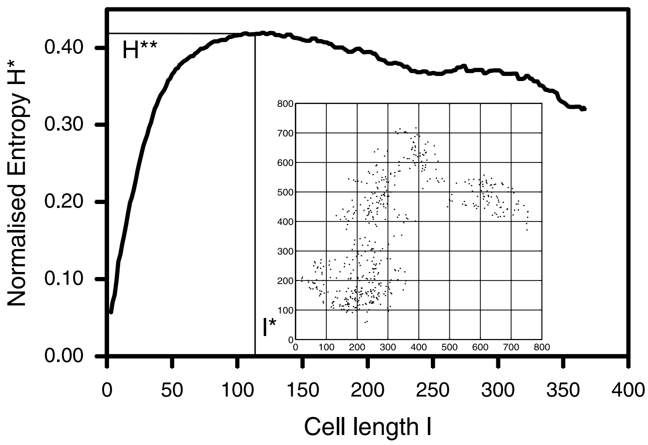

Figure 1. Configurational entropy H^* as a function of cell length l for a fractal point distribution (inset, arbitrary length units). To allow comparison of entropy values H at different scales, H has been normalised to H^* by its maximum value possible at each scale. Indicated are H^{**}, the maximum of H^* over all scales (shown up to $\approx L/2$), and l^*, the optimum length, at which H^{**} occurs. Our main interest will be the temporal evolution of H^{**}.

of a 2D site-percolation model [e.g., *Sahimi*, 1994]. For this, a square matrix of blocks is considered, where each block is either permeable or impermeable with a certain probability for permeability p, $p \in [0, 1]$. Because of the randomness of the resulting distribution, percolation is clearly a statistical problem, i.e., a universal testbed. For each microscopic probability p exists a macroscopic probability P, that the matrix is permeable as a whole. P is found to be very small if $0 < p < p_c$, where p_c is called critical probability for the percolation threshold, it grows close to unity if $p_c < p < 1$. Numerical simulations determine the critical permeability to be $p_c = 0.5928$ [e.g., *Stauffer and Aharony*, 1992]. The percolation threshold is a frequently used example for a critical point. The size of the largest cluster of connected permeable blocks as a function of probability p is one example for power law scaling at the critical point. In the context of seismicity, *Morein et al.* [1997] found a simiiliarity between the number-size statistics of clusters in the slider-block model used for simulations of earthquakes and of the site-percolation model. As shown in Figure 2, configurational entropy is an appropriate quantity for the detection of the critical point at $p_c = 0.5928$ as it undergoes its maximum just at the percolation threshold, independently of the size of the array. Note the following slow decline of H^{**}. Our method simultaneously determines the entropy maximum and the optimum length l^*. The possible broadness of the entropy maximum hump (cf. Figure 1) sometimes makes the exact determination of l^* difficult, however, so that we refrain from using it here. This is somewhat unfortunate as l^* would be a very obvious and plausible measure of correlation length. Examples of l^* curves may be found in *Goltz* [1997], however.

4. Results for Real World Data

[5] To test our approach with real world data we examined seismicity of Southern California. For completeness of

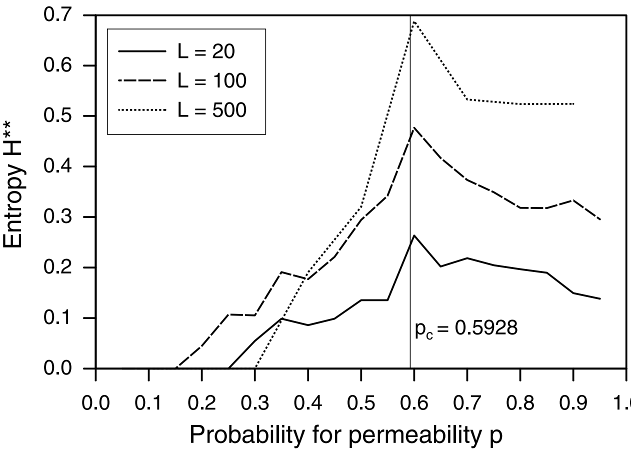

Figure 2. Detection of the phase transition in the case of a 2D site-percolation model using entropy H^{**}. Progressing towards the critical point, a significant increase in entropy is observed. H^{**} peaks exactly at the critical probability for the percolation threshold and then slowly declines. The result is widely independent of overall grid size L (arbitrary length units).

the catalogue we used data from January 1984 to December 2000, selecting only events of magnitude $M \geq 1.8$. Taking into account that seismic activity is a discontinuous process and that H^{**} depends on the number of data points, we used a fixed number of events (n_{ev} = 350) for each temporal window. The range of cell sizes l was about 5 km to about 100 km for each window. Figure 3 (middle) shows H^{**} from January 1988 to December 1997 in the vicinity of the 1992, M7.3 Landers earthquake as an example: the size of the investigated area centered on the epicentre is $L \times L$ = 220 $\times$ 220 km^2, containing about 35,000 events and resulting in about 700 windows using an overlap of 300 events. As there are no internal free parameters such as e.g., weights or fitting coefficients in the analysis algorithm itself, the result is independent of algorithmic details. Varying n_{ev} and the overlap within useful bounds results in a change of the overall level of H^{**} but the shape is preserved. Concerning external parameters, fluctuations were found to be independent of the choice of L for 165 to at least 280 km. Imposing a sensible upper magnitude cutoff only removes a minor part of events and has no pronounced effect on the shape of H^{**}. For the example concerned we are thus confident that the result is robust. There is an apparent correlation between H^{**} and seismicity. Entropy maxima are found at the times of occurrence of Joshua Tree (JT) (April 22, 1992, M6.1) and Landers (LD) (June 28, 1992, M7.3). Furthermore, the result nicely agrees with what was obtained for the percolation data in that there are clear differences before and after the catastrophe. The agreement is especially strong in the "aftershock regime" while the acceleration phase is much clearer in the synthetic data. Entropy fluctuations are of the order of ±20%, comparable to but more pronounced than the results of *Main and Al-Kindy* [2002] which were obtained for global seismicity. Figure 3 (top) also gives H^{**} for a temporally randomised version of the data which was obtained by creating permutations without replacement [cf. *Schreiber and Schmitz*, 2000]. None of the features observed in the original data survives and the fluctuations are much smaller. We can reject the null hypothesis that the original entropy fluctuations of ±20% are due to random chance with 99% confidence as the maximum fluctuation found in 99 different realisations of randomised catalogues is about ±6%. Further evidence for the physical significance of our result comes from the fact that the Hurst coefficient for the

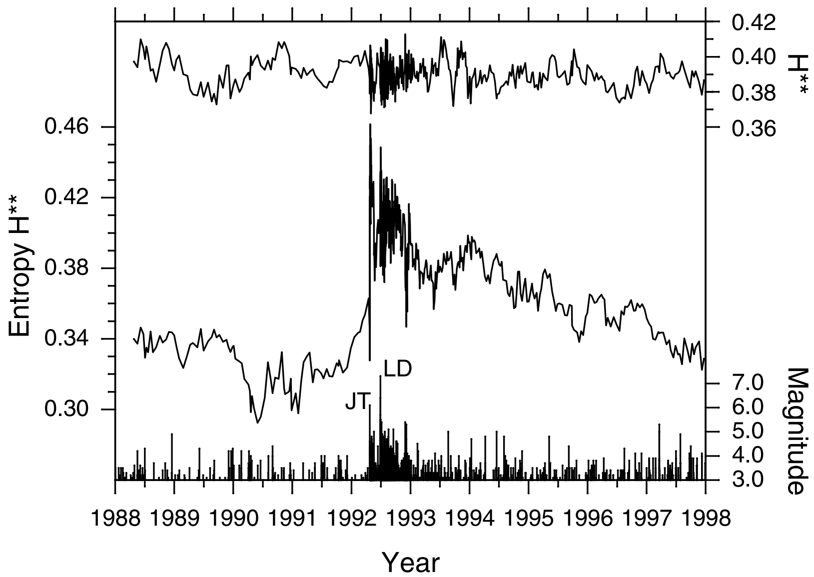

Figure 3. Entropy H^{**} for the earthquake population of a 220 $\times$ 220 km^2 region centered on the 1992, M7.3 Landers (LD) event (middle) and of the temporally randomised catalogue (top). Also shown is the magnitude history of the region (bottom). The time period is January 1988 to December 1997. One may note an accelerated increase in H^{**} prior to the two main events followed by a slow linear decline in the original data while no such features are apparent in the surrogate data. Note the equal scales of the two entropy curves.

original data as obtained from *R*/*S*-statistics [e.g., *Goltz*, 1997] is about 0.59 while the randomised curve of Figure 3 has 0.42. The original value indicates persistance as expected after a main shock due to slow aftershock declustering while the lower value indicates loss of memory due to randomisation. Appropriate numerical simulations (not shown) confirmed that our method is indeed highly sensitive to the degree of clustering. With our one example we have no means of assessing the significance of the apparent pre-seismic increase of H^{**} despite the fact that we have not observed any excursion of this magnitude in any of the random series. The real question is whether it would be possible to unambiguously discern the increase when monitoring H^{**} without a priori knowledge of an impending earthquake. Thus, while we believe that the increase is real in our example, we by no means claim that it constitutes a generally useful precursor. In fact, a cursory search has shown that a discernible pre-seismic increase cannot be reproduced for all major earthquakes while the post-seismic slow decline seems robust.

5. Conclusions and Outlook

[6] Analysis of seismicity in the vicinity of the Landers earthquake and other regions as well as synthetic point distributions confirmed that configurational entropy can be a powerful approach for the characterisation of complex earthquake population dynamics. The critical point hypothesis predicts a divergence of characteristic lengths. Our analysis of a 2D site-percolation model shows agreement with theory as entropy undergoes its maximum exactly at the percolation threshold p_c. Furthermore, the observed entropy curve for real data is described perfectly by what is predicted by the percolation model. Taken together with statistically significant entropy fluctuations of ±20% we conclude that the Earth's crust is in a state of intermittent criticality. While we believe that the observed pre-seismic increase of entropy is real we have no means to assess its significance as a precursor at the current stage. Our result should warrant a systematic spatio-temporal test. The sensitivity of H^{**} to clustering naturally leads to the question if our approach might be useful in achieving the largely unreached goal of separating correlated from uncorrelated earthquakes. As all conventional declustering methods require the definition of internal parameters as e.g., predefined space-time windows [e.g., *Goltz*, 2001] our approach has the clear advantage of no internal parameters at all. We therefore propose to include configurational entropy in the process of earthquake declustering, at least for testing the results of other methods, and will adress this issue in a future paper.

[7] **Acknowledgments.** CG's work was partly supported by Science and Technology Agency of Japan fellowship no. 399002. The earthquake data was obtained from the Southern California Earthquake Data Center (SCEDC).The paper benefitted greatly from a review by I. Main and V. Kossobokov. This is publication no. 16 of the SFB 574 at the University of Kiel.

References

Bak, P., and C. Tang, Earthquakes as a self-organized critical phenomenon, *J. Geophys. Res.*, *94*, 15,635–15,637, 1989.

Bak, P., K. Christensen, L. Danon, and T. Scanlon, Unified scaling law for earthquakes, *Phys. Rev. Let.*, *88*, 178–501, 2002.

Beghdadi, A., C. Andraud, J. Lafait, J. Peiro, and M. Perreau, Entropic and Multifractal Analysis of Disordered Morphologies, *Fractals*, *1*, 671–679, 1993.

Boltzmann, L., Einige allgemeine Sätze über Wärmegleichgewicht, *Sitzungsber. Kais. Akad. Wiss. Wien.*, *63*, 679–711, 1871.

Bowman, D. D., G. Ouillon, C. G. Sammis, A. Sornette, and D. Sornette, An observational test of the critical earthquake concept, *J. Geophys. Res.*, *103*, 359–372, 1998.

Bufe, C. G., S. P. Nishenko, and D. J. Varnes, Seismicity trends and potential for large earthquakes in the Alaska-Aleutian region, *Pure Appl. Geophys.*, *142*, 83–99, 1994.

Clausius, R., On the motive power of heat, and on the laws which can be deduced from it for the theory of heat, *Ann. Physik*, *LXXIX*, 368–500, 1850.

Dodge, D. A., G. C. Beroza, and W. L. Ellsworth, Detailed observations of California foreshock sequences: Implications for the earthquake initiation process, *J. Geophys. Res.*, *101*, 22,371–22,392, 1996.

Ellsworth, W. L., A. G. Lindh, W. H. Prescott, and D. H. Herd, The 1906 San Francisco earthquake and the seismic cycle, in *Earthquake Prediction: An International Review, Maurice Ewing Ser., 4*, edited by D. W. Simpson and P. G. Richards, 126–140, AGU, Washington, D.C., 1981.

Fedotov, S. A., Regularities of the distribution of strong earthquakes in Kamchatka, the Kurile Islands, and northeast Japan, *Tr. Inst. Fiz. Zemli, Akad. Nauk SSSR*, *36*, 66–93, 1965.

Goltz, C., *Fractal and Chaotic Properties of Earthquakes*, Lecture Notes in Earth Sciences, Springer-Verlag, Berlin, 1997.

Goltz, C., Decomposing spatio-temporal seismicity patterns, *Natural Hazards and Earth System Sciences*, *1*, 83–92, 2001.

Jaume, S. C., and L. R. Sykes, Evolving towards a critical point: A review of accelerating seismic moment/energy release prior to large and great earthquakes, *Pure Appl. Geophys.*, *155*, 279–305, 1999.

Jones, L. M., and P. Molnar, Some characteristics of foreshocks and their possible relationship to earthquake prediction and premonitory slip on faults, *J. Geophys. Res.*, *84*, 3596–3608, 1979.

Keilis-Borok, V. I., The lithosphere of the Earth as a nonlinear system with implications for earthquake prediction, *Rev. Geophys.*, *28*, 19–34, 1990.

Knopoff, L., T. Levshina, V. I. Keilis-Borok, and C. Mattoni, Increased long-range intermediate-magnitude earthquake activity prior to strong earthquakes in California, *J. Geophys. Res.*, *101*, 5779–5796, 1996.

Main, I. G., and F. H. Al-Kindy, Entropy, energy, and proximity to criticality in global earthquake populations, *Geophys. Res. Lett.*, *29*, 2002.

Mogi, K., Some features of recent seismic activity in and near Japan, (2) Activitiy before and after large earthquakes, *Bull. Earthquake Res. Inst., Tokyo Univ.*, *47*, 395–417, 1969.

Morein, M., D. L. Turcotte, and A. Gabrielov, On the statistical mechanics of distributed seismicity, *Geophys. J. Int.*, *131*, 552–558, 1997.

Sahimi, M., *Applications of Percolation Theory*, Taylor & Francis, 1994.

Sammis, C. G., and S. W. Smith, Seismic cycles and the evolution of stresscorrelation in cellular models of finite fault networks; Seismicity patterns, their statistical significance and physical meaning, *Pure Appl. Geophys.*, *155*, 307–334, 1999.

Sammis, C. G., D. Sornette, and H. Saleur, Complexity and earthquake forecasting, Reduction and Predictability of Natural Disasters, *SFI Studies in the Science of Complexity, XXV*, edited by J. B. Rundle, W. Klein, and D. L. Turcotte, 143–156, Addison-Wesley, Reading, Mass., 1996.

Schreiber, T., and A. Schmitz, Surrogate time series, *Physica D*, *142*, 346–382, 2000.

Shannon, C. E., A mathematical theory of communication, *Bell System Technical Journal*, *27*, 379–423 and 623–656, 1948.

Sornette, A., and D. Sornette, Earthquake rupture as a critical point: Consequences for telluric precursor, *Tectonophysics*, *179*, 327–334, 1990.

Sornette, D., and C. G. Sammis, Complex critical exponents from renormalization group theory of earthquakes: Implications for earthquake predictions, *J. Phys. I France*, *5*, 607–619, 1995.

Stauffer, D., and A. Aharony, *Introduction to Percolation Theory*, Taylor & Francis, London, 1992.

Sykes, L. R., and S. Jaume, Seismic activity on neighbouring faults as a long-term precursor to large earthquakes in the San Francisco Bay area, *Nature*, *348*, 595–599, 1990.

Wyss, M., K. Shimazaki, and T. Urabe, Quantitative mapping of a precursory seismic quiescence to the Izu-Oshima, 1990 (M6.5) earthquake, Japan, *Geophys. J. Int.*, *127*, 735–743, 1996.

Zöller, G., S. Hainzl, and J. Kurths, Observation of growing correlation length as an indicator for critical point behavior prior to large earthquakes, *J. Geophys. R.*, *106*, 2167–2175, 2001.

C. Goltz and M. Böse, Department of Geophysics, Otto-Hahn-Platz 1, 24118 Kiel, Germany. (goltz@geophysik.uni-kiel.de; mboese@geophysik.uni-kiel.de)